Exercise 3A

1 $(1\times 6)+(4\times 3)+(3\times 2)+(2\times 4)=\bar{x}(1+4+3+2)$ Use $\sum m_1 x_1 = \bar{x}\sum m_1$

$6+12+6+8=10\bar{x}$ Simplify.

$32=10\bar{x}$

$3.2=\bar{x}$ Solve for $\bar{x}$.

Centre of mass is at (3.2, 0). Give both coordinates.

2 $(1\times 2)+(2\times 5)+(3\times 1)=\bar{y}(1+2+3)$ Use $\sum m_i y_i = \bar{y}\sum m_i$

$2+10+3=6\bar{y}$ Simplify.

$15=6\bar{y}$

$2.5=\bar{y}$ Solve for $\bar{y}$.

Centre of mass is at (0, 2.5). Give both coordinates.

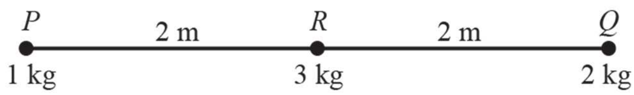
3 $(2\times -1)+(3\times -4)+(5\times 5)=\bar{x}(2+3+5)$ Use $\sum m_i x_i = \bar{x}\sum m_i$

$-2+-12+25=10\bar{x}$

$11=10\bar{x}$ Simplify.

$1.1=\bar{x}$ Solve for $\bar{x}$.

Centre of mass is at (1.1, 0). Give both coordinates.

4 

Draw a diagram. The rod has no mass.

$(1\times 0)+(3\times 2)+(2\times 4)=\bar{x}(1+3+2)$ Take P as the origin and use $\sum m_i x_i = \bar{x}\sum m_i$

$0+6+8=6\bar{x}$ Simplify.

$\frac{7}{3}=\bar{x}$ Solve for

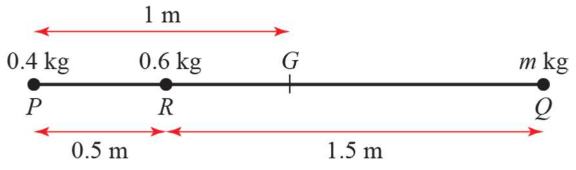
$PG=2\frac{1}{3}\text{ m}$

5 $(5\times 4)+(3\times 2)+(m\times 5)=4(5+3+m)$ Use $\sum m_i y_i = \bar{y}\sum m_i$

$20+6+5m=32+4m$ Simplify.

$m=6$ Solve for m.

6



The rod, being light, has no mass.

Draw a diagram showing all the information. G is the centre of mass. Assume the mass of the particle required is m kg.

Take P as the origin.

$$(0.4\times 0)+(0.6\times 0.5)+(m\times 2)=1\times(0.4+0.6+m)$$

Use $\sum m_i x_i = \bar{x}\sum m_i$

$$0.3+2m=1.0+m$$

Simplify.

$$m=0.7$$

Solve for m.

The mass of the particle is 0.7 kg.

7 $$(2m\times a)+(3m\times 2)+(7m\times -1)+(8m\times 1)=1\times(2m+3m+7m+8m)$$

Use $\sum m_i y_i = \bar{y}\sum m_i$

$$2ma+6m-7m+8m=20m$$

Divide by m.

$$2a+7=20$$

$$a=6\frac{1}{2}$$

Solve for a.

8 Suppose the particle is placed at $(0, y)$.

$$(3\times -2)+(2\times 7)+(1\times 4)+(6\times y)=0\times(3+2+1+6)$$

Use $\sum m_i y_i = \bar{y}\sum m_i$

$$-6+14+4+6y=0$$

Simplify.

$$6y=-12$$

$$y=-2$$

Solve for y.

The particle must be placed at (0, –2).

Give both coordinates.

9 $$5+m_1+m_2=10$$

Use the total mass.

$$m_1+m_2=5 \qquad \textbf{(1)}$$

$$(5\times 2)+(m_1\times 3)+(m_2\times -2)=1\times 10$$

Use $\sum m_i x_i = \bar{x}\sum m_i$, and $m_i = 10$.

$$10+3m_1-2m_2=10$$

Simplify.

$$3m_1-2m_2=0 \qquad \textbf{(2)}$$

Adding $\textbf{(2)}+2\times\textbf{(1)}$, $2m_1+2m_2=10$

$$5m_1=10$$

Eliminate m_2

$$m_1=2$$

Solve for m_1

$$m_2=3$$

Use **(1)**.

10 Let M be the total mass of the system, so we have

$M = (m-1)+(5-m)+m+(m+1)$

i.e. $M = 2m+5$

Given that the centre of mass is at $(0,1)$ taking moments gives

$-1\times(m-1)+(5-m)+2m = M$

i.e. $M = 6$

Hence $6 = 2m+5$ so $m = 0.5$

Challenge

Without loss of generality we can assume that $P=(0,0)$, $Q=(2,0)$ and $R=(5,0)$

Then the total mass is $M = 1+2+3 = 6$

Let $G=(x,0)$ be the centre of mass then taking moments gives

$6x = 2\times 2+3\times 5 = 19$ i.e. $x = \dfrac{19}{6}$

Hence the ratio

$PQ:PG = 2:\dfrac{19}{6} = 12:19$